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COMPUTABLE MODELS FOR EXPLORING THE IDEA OF THE "BALANCE OF POWER"

A DISSERTATION SUBMITTED TO

THE FACULTY OF THE DIVISION OF THE SOCIAL SCIENCES

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BY

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INTRODUCTION

Since at least 1700[1] the term "balance of power" has implied an attempt to illuminate the international system by simple abstract models. Mechanical metaphors and hydrostatic analogies with trimming a ship should, of course, be discounted as convenient expressive shorthand for their implicit models. Confusion has persisted,[2] and the ultimate importance of the subject matter makes it worthwhile to explore, by any means, the logic and formal structure of arguments and models that might be used to describe or prescribe for international systems.

My approach is to spell out in computable mathematical models various relationships suggested primarily by these metaphors and secondarily by the histories of the periods to which they have been applied.[3]

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[1]J. Swift, "A Discourse of the Contests and Dissensions between the Nobles and the Commons in Athens and Rome," The Works of the Rev. Jonathan Swift, D.D., Vol.III (London: J.Johnson; J.Nichols and Son; et.al., 1808), pp.7-8. First printed for J. Nutt, in the year 1701.

[2]See E. B. Haas, "The Balance of Power: Prescription, Concept, or Propaganda," International Politics and Foreign Policy: A Reader in Research and Theory, ed. J. W. Rosenau (New York: The Free Press of Glencoe, Inc., 1961), p.324, for a critical account.

[3]Other dissertations in progress from the Ford Workshop in International Politics make systematic attempts to describe verbally those models especially relevant to particular historical systems, e.g. quattrocento Italy (Winfried Franke's dissertation in progress on the Italian city-state system, University of Chicago). My work benefits only casually from cross-fertilization with the Workshop's empirical studies. It is principally confined and directed by opportunities to spell out ideas in the simplest mathematical form.

I call such a concrete mathematical expression of possibly relevant relationships a realization-model.[4] There can be many such, even for very simple looking verbal models. All my work does not begin to exhaust the possibilities which can be read out of and into Kaplan's "balance of power" model.[5] The advantage of mathematical models over ambiguous and suggestive verbal models is that their behavior is perfectly well defined. One can progressively alter the conditions, explore the subsequent changes in model behavior and keep an exact account of these effects. Words, on the other hand, are more slippery and verbal models can only too easily be reshaped and reapplied with scarcely any formal record of the difference.

This dissertation describes two models I have made and used for exploring the idea of the "balance of power."

The pilot model of chapter i was designed to prove a computer model possible. In exploring it, the most interesting hint found was a reminder that the balanced system may well maintain itself by highly

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[4]I speak of realization-models rather than simulation models because I make a virtue out of the necessity that one proceed by partial stages. Not only are many relevant details of an empirically applicable idea left out of a realization-model, some of the realization-model's few details may be superfluous in a particular context. My ambition is to explore and build theory piecemeal, not entire. For this it is even sometimes very useful to explore counterfactual or preposterous assumptions to get the hang of the logic.

My realization-models, then, are something doubly less than attempted simulations of the international system (which would ideally obviate the need for a theoretical account apart from the computer representation). Not only are they incomplete representations of the "balance of power" idea, they do not supplant verbal theory. For the foreseeable future they only give advice on how theory, today verbal, someday mathematical, might be put together.

[5]M. Kaplan, System and Process in International Politics (New York: John Wiley and Sons, 1957), p.23f. *et passim.*

unbalanced wars (against the top dog) because such Grand Coalitions have a high short term excess of gains over losses.

The minimal model of chapter ii was designed to reconcile the divergent insights of Kaplan and Arthur Burns[6] into the way numbers affect the stability of the system. Some clarification of this difference has been obtained as a co-product with the minimal model. It itself suggests absurdly simple strategic reasons, quite apart from balancing roles and tactical possibilities, why larger systems (five actors, say) should be stabler.

The two models, one a computer model puttered with in ignorance, the other a pencil and paper simplification to the barest essentials, may be taken as paradigms and basis for further work.

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[6] A. Burns, "From Balance to Deterrence: A Theoretical Analysis," <u>World Politics</u>, IX, 4 (July, 1957), 494-529.

CHAPTER I

PILOT COMPUTER MODEL

The problem answered by this chapter is the feasibility of a computer model for exploring the "balance of power." The pilot model which does this job is a computer simulation of the play of a simple diplomatic game.

Firstly, the model was designed to prove the possibility of a computer realization-model for the "balance of power." Earlier researchers had tried unsuccessfully to design one.[1] By realization-model I mean a partial representation, where less useful details are suppressed if possible. The minimum is a concrete system of n actors capable of inflicting damage on each other's relative capabilities and behaving in patterns not clearly irrelevant to the "balance of power" idea.[2]

Secondly, to be of any interest, the model had to be used to find some theoretical arguments that not everyone takes as a matter of course. The first fruits were bound to look commonsensical in hindsight. It will not always be so; longer chains of artificial reasoning will have a lastingly technical air.[3]

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[1]M. Kaplan, A. Burns, and R. Quandt, "Theoretical Analysis of the 'Balance of Power,'" Behavioral Science, V, No.3 (July, 1960), 240-252.

[2]I take Kaplan's essential rules and other discussion of the "balance of power" as a further description of that idea, System and Process . . . , p.23 et passim.

[3]Cf. parts of the next chapter, such as the argument about how secondary dilution offsets alternation, below, pp.64ff.

The proof case is the use of the pilot model to controvert a bland assumption virtually written into its own design. This assumption that balanced systems tend to have "balanced," that is, evenly matched, wars has been made by established writers in the field. "[S]elf interest . . . produce[s] very nearly evenly matched coalitions."[4] The pilot model suggests instead that Grand Coalitions can well occur because they appeal to actors' desire for short term gains. The slight sacrifice of directing them against the largest actor makes all the necessary difference to the long run. The most interesting aspect of this simple result is that it followed naturally from the workings of a mathematical model, when the expectations of the designer visible leant the other way. Merely verbal theorizing might have managed to miss the point and preserve the assumption.

Other interesting points, including a hint about taking the risk out of utility theory, emerge as byproducts.

Background

Before 1959, Kaplan, Burns and Quandt desired to make what I call a realization-model for exploring the "balance of power." In their own words, such an object is to be used "not as itself a model, but as a device for 'playing out' models and theories."[5] Their device was a game to be played by humans. Wanting controlled experiments, their first choice would have been to have a machine play the game, because

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[4]K. Deutsch and J. Singer, "Multipolar Power Systems and International Stability," World Politics, XVI, No.3 (April, 1964), p.403. It is also evident that Burns, "From Balance . . . ," pp.494-529, and W. Riker, The Theory of Political Coalitions (New Haven: Yale University Press, 1962), pp.247-278 think about more evenly matched wars rather than about Grand Coalitions.

[5]Kaplan, Burns, and Quandt, "Theoretical Analysis . . . ," p.240.

it is difficult to prescribe consistent styles of play to humans.[6]

As Quandt and the others have told me, the perceived obstacle to a computer model was the variety of alliance patterns. Five distinct actors, for example, have a choice among ninety different two-sided wars--ninety-one if one includes the important case where no-one fights at all. They did not think it possible or desirable to develop ad hoc rules to make such choices.

The principal results of this chapter suggest a simplification of their choice. If there is a war at all, either everyone gangs up for a short war against the largest actor, which is not destabilizing, or upon the smallest actor, which is destabilizing.[7] From history, however, one expects the type of wars in the "balance of power" model to cover a wider range of behavior than this. They would have had to find qualifications for that first choice. My pilot model is also a way for finding such qualifications.

In origin, my pilot model is the computer model Quandt wanted to make.

The trick of the game in my pilot model is essentially to take the players of Burns' game[8] out of face-to-face contact and to make them play the game through independently dated bids.

By such control of the bargaining process researchers could control, more or less, human players' mutual information and incentives toward alliance. When nobody knows his name, a player's cultural

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[6] Ibid., p.245.

[7] Kaplan and I customarily measure instability by the loss of an actor.

[8] Kaplan, Burns, and Quandt, "Theoretical Analysis . . . ," pp.247ff.

attributes and his behavior in other rounds of the game can be kept secret from other players considering him for alliance. Instead, players can be made to take into account only each others' military-economic size and alliances presently formed or formable for a round of war. The isolation and control of such variables is of evident importance to a "balance of power" model.

To obtain from humans conservative or aggressive styles of play--not very reliably--a researcher tinkers with the payoffs and other rhetorical items of the experimental setting. Happily, the feature of play through dated bids so simplifies a competitive game that a machine can play it for all the actors, and a machine, of course, will do just what it is told.

Game

Figure 1 is the flow chart of my simulable diplomatic game. It should be distinguished from Fig.2,below p.37,which is the flow chart of a computer model simulating the play of the game. Like Burns' game, just cited, this one proceeds by rounds of war, each preceded by bargaining. My war game happens to be a much more stripped down version than Burns', but this is a minor matter. More important is my imposition of such a structure on the commitment opportunities of the actors that bargaining may be made by a simulable system of dated bids. The game of Figure 1 is an onion with three laters: military exchange, sequence of "diplomatic events," and dating of individual bids by actors.

Military Exchange

The following are practically the simplest reasonable rules for military exchange between two coalitions (sides) which take account of the members' several military-economic SIZEs. In a war between two

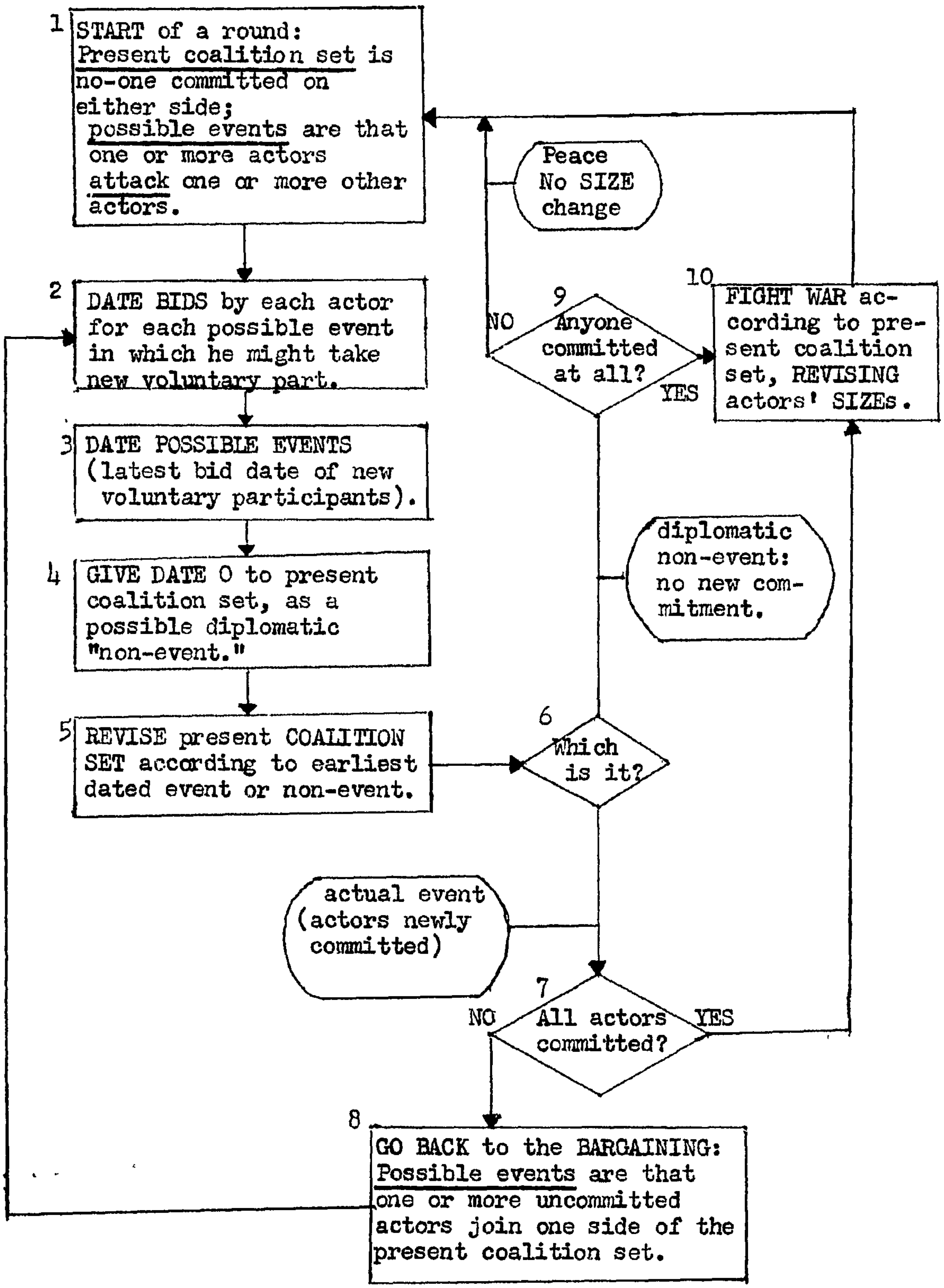

Fig.1--Simulable diplomatic game

sides, each side simultaneously inflicts upon the other side a fraction of the inflicter's total SIZE. Of these inflicted losses a fraction is gained by the inflicter as booty. Denoting total prewar SIZEs of Red and Blue sides as R and B, respectively, one has the following symmetric formulae.

$$\text{(Eqs.1)} \quad \begin{aligned} \text{postwar } R &= R + (\text{booty factor}) \times R - (\text{loss factor}) \times B \\ \text{postwar } B &= B + (\text{booty factor}) \times B - (\text{loss factor}) \times R \end{aligned}$$

It will be convenient to denote by F the fraction of captured goods which are usefully reappropriated. This parameter, which is, by definition, the booty factor divided by the loss factor, may be named durability ratio. (It later proves to affect this chapter's principal result; high durability makes evenly divided wars economically feasible.)

The above formulae may produce "overkill." The simple thing to do in such case is to scale down the action to simple annihilation. Thus, if (Eqs.1) give a negative postwar R, use instead:

$$\text{(Eqs.1')} \quad \begin{aligned} \text{postwar } R &= 0 \\ \text{postwar } B &= B + \frac{R(FB-R)}{B-FR} \end{aligned}$$

In every case, members of a coalition (side) share their gains and losses proportionally to their SIZEs.

The foregoing describes what happens inside Box 10 of the flow chart in Figure 1: "FIGHT WAR according to present coalition set, REVISING actors' SIZEs." Coalition set is simply my term for a partition of the actors into three camps: red, blue and neutral.

Diplomatic Events

The flow chart itself represents the formation of these coalition sets by one or more passes through the blind bargaining subgame, Boxes 2 through 5 (described below). Each such pass corresponds to a diplomatic event, which is the firm commitment of actor(s) to fight in the impending war. On the first pass, one or more actors may declare war on one or more other actors (cf. Box 1). In later passes, one or more actors may join one of the sides already formed (cf. Box 8). This opportunity is re-extended to the uncommitted actors each time a new commitment is made (cf. route:Box 6 to Box 7 to Box 8). Thus, n actors can draw out the bargaining process to as many as n-1 distinct diplomatic events: attacker and victim are committed on the first pass; the rest join in one by one. (This is what actually happened in the pilot model simulation of the game.)

The process used in each pass, Boxes 2 through 5, is of some interest. I call it a blind bargaining game.[9] A simplified application will introduce it. Suppose some players and a pie to be distributed

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[9] This game is bargaining stripped down to two truisms. First, apart from tactical gambits, all normal bargainers make their less preferred bids later. Second, the timing of their bids contains an autonomous factor, independent of momentary dickerings.

A local camera shop once provided a clear example of actors bidding without any dickering and with a convenient display of one actor's declining price schedule. In the window were a camera and the announcement that its price would come down a dollar a day or something to that effect. Probably this "Dutch Auction," as the shop called it, would have been the best hint for making the pilot model. I myself had already gotten the hint from Henry VIII. In the 1530's Henry was contemplating alliance with the Emperor, Charles V, against France. Henry's instructions to his ambassador at the court of Charles V contain a list of about ten successive prices which the ambassador is to try to exact for an English alliance. They begin with the French crown itself, run on through the Duchy of Normandy and other less desirable parcels of real estate, and finish with a modest cash subsidy. State Papers of the Reign of Henry VIII, 11 Vols. (London: by the Commissioners for Printing and Publishing State Papers, 1830-1852).

This document and impressions of R. Bellman's Dynamic Programming (Princeton, N.J.: Princeton University Press, 1957) inspired my simulation model.

In a similar computer model context, Bellman teaches the principle of deciding one step by first hypothetically deciding all possible subsequent steps. That is what my simulated actors have to do in order to date their bids in any one bargaining subgame.

(one of a finite number of ways) according to the will of any majority. What is a bargaining method which contains a minimum of present negotiation? Surely this. For each possible division of the pie each player shall make just one bid, writing down the date at which he plans to agree to that division (if the matter is still undecided). He will be bound to leave open any offer once made, but until a decision is made, he is free to make additional contrary offers. The first division to gather the agreement of a majority is the one made.

In the present application of the blind bargaining game, instead of pie distributions, the actors bid for various possible diplomatic events, that is, either a declaration of attack or a declaration of joining. The effective set, on all of whose members' agreement a possible event waits, is instead of a variable majority, precisely the set of attacking or newly joining actors. The effective date of an event is , again, the date at which the latest member of the effective set makes his bid (Box 3).

Only new commitments have been defined to be diplomatic events. In the present application it is natural to give separate mention to the impasse solution of the bargaining subgame. Just as wage negotiations which go to a deadline without a settlement are supposed to produce a strike, so diplomatic bargaining which produces no new commitment should be closed off by war among the hitherto committed actors, if any. This is the meaning of the diplomatic "non-event" of Box 4 and exiting from Box 6 to Box 9 in the chart.

If not a deadline, some more complicated stop rule would be necessary to force or recognize the issue of a bargaining situation. Note, however, that a definite starting point to the bargaining period is not a logical necessity. Writing dates on the bids instead of bidding in real time brings this simplifying possibility to light.

Dating Bids

The game is now reduced to the dating of bids. For the players, other rules are explanatory, not operational. An experimenter might rearrange the actual military and diplomatic events and administer questionnaires to isolated subjects. A sample questionnaire could begin thus.

> You are player A in a system of SIZEs where A=10, B=7, C=12, D=5, E=13 (here might follow some past history or some rhetorical matter, such as "E has been making unfavorable comments on your religion");
> C has declared war on D.
> Here are the things you can possibly do. How early are you amenable to each one?
> Write a number in the space for any event you prefer to letting C and D have their war.
>
> a. Join C ______
> b. Join D ______
> c. Join C together with E ______ , etc.

Intelligent play of this game presupposes that in dating a bid an actor answers a number of private questions which relate to how much a course of action will tend to increase his satisfaction with the postwar distribution of SIZEs.[10]

I define an actor's <u>style</u> of play to be a function which assigns a value (satisfaction) to every such possible distribution of SIZEs. The rationale of the computer simulation is to turn such styles into dates on bids, hence into the action of the system. The general rule is that an actor bids before the deadline for other events (only if and) in proportion as he prefers their consequences to the consequences of war fought out by the present coalition set. It breaks down into giving

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[10] The choice to analyze by means of prediction to immediate postwar satisfactions requires a section of comment, see page 30 below.

more or less reasonable answers to the following five questions an actor might ask himself in dating his bids for a blind bargaining subgame.

1. What postwar distribution of SIZEs results from war fought out by a given coalition set?

2. If a particular diplomatic event happens now, what further diplomatic events will happen? i.e. What coalition set will ultimately fight?

3. In a (postwar) SIZE distribution what relative weight shall I give to the balancedness of the (foreign) part of the system as compared with my own SIZE?

4. How do I measure foreign imbalance?

5. The deadline apporaches and I do not have the alliance I would most like. Shall I settle for a second best alliance?

Since I want to discuss the implications of the simulation rationale piecemeal, the simulation's behavior will be described in the next section and the rationale after. All that is prerequisite to stating the behavior is reference to the central question. Each actor's style of play is especially characterized by a personal parameter, his <u>co-efficient of disutility for foreign imbalance,</u> acronym CDFI. This parameter measures the relative importance an actor places on equalizing the other actors as compared with growing or maintaining his own SIZE. Thus, by definition, CDFI measures an actor's commitment to (immediate short-run) balance-keeping.

Computer Model Performance

A computer simulation[11] of the play of the game of Figure 1 was run on MANIAC III. Apart from a few preliminary runs which will be briefly mentioned later (page 17 below), the runs made fall into three families (zero, one or two deviant actors, as defined below). Most runs

[11] Flow chart and algorithms are given below, pages 37-41.

were made in the first family; fewest in the third. Also, most of the usable results (those which can be given an unartificial explanation) belong to the first family and fewest to the third.

Input

In all runs a booty factor of 10 per cent and a loss factor of 15 per cent were used.

All runs have five actors, initially of equal SIZE.

The three families of runs are distinguished by whether zero, one or two of the five actors are deviants. Deviants are defined as actors with zero CDFI, that is, no commitment to balance-keeping.

The non-deviant actors in any one run have a common positive CDFI. The size of this common CDFI is the parameter indexing the family of runs.

Output

It was only to be expected that increasing the CDFI increases the stability of the system, and it does. Cross-comparison of the families and details of the behavior are of some interest.

At the least detailed level, Family A (Common CDFI), confirms the expected truism by manifesting three types of behavior, of ascending stability.

> Type One--for low common CDFI--Action (war) to elimination of an actor.
>
> Type Two--for intermediate common CDFI--Continuing action without elimination of any actor. (This is the noteworthy type).
>
> Type Three--for high common CDFI--Inaction (peace).

Round by round details of who fights whom

Type One--In the first round four actors make war on a fifth. In subsequent rounds they finish him off.

Type Two--Again, four actors make war on a fifth in the opening round. In the second round, however, a different actor is made victim of a four actor coalition, and the other three are successively made victim in the third, fourth and fifth rounds. By this point all the actors' SIZEs have become unequal, and forever[12] after the largest actor is the victim.

Type Three--No details.

Bargaining sequence

The details of the bargaining sequence boil down to this. In no blind bargaining subgame of this or any other family did two actors make a joint commitment. Since only one actor commits himself each time, the bargaining of every round is drawn out to four diplomatic events. In the subgame for each such event, it is the largest not-yet-committed non-victim who commits himself.

Since the simulation makes such a meager use of the blind bargaining subgame, I prefer to suppress the details of the bargaining sequence and use the previous descriptions where possible. The realization model-simulation model distinction is a way of saying that one can and should suppress details in the computer model and/or theoretical model to find a useful level of comparison.

In family B (one deviant) the same three types of behavior occurred as in the previous family of runs. The two boundary values of the parameter (the common CDFI of the four non-deviants) which separate Type Two behavior from Types One and Three are higher than in the

[12]Once, when I suppressed all the details, real and hypothetical, of the bargaining sequence, which took two pages per round to print, the computer produced a run of over 275 rounds of this sort in half an hour.

previous family (where all five actors were non-deviant).

In the new Type One runs, namely runs that would have been stable but for the one actor's deviancy, the deviant was not the actor eliminated. Neither was he the largest survivor. Typically he is the second largest actor, but this proves sensitive to a rather insignificant cause.[13]

In Family C (two deviants) the boundary CDFI are still higher and deviants again are neither victim nor largest survivor.

From the performance I select for especial discussion the fact that all wars were Grand Coalitions of four actors against one rather than evenly divided wars such as three against two.[14]

Topics in the Simulation's Rationale

In this section I discuss the pilot model's implicit answers to the five questions an intelligent actor was said[15] to ask himself in dating his bids. First, however, I note an omitted question: "What are they doing over there without me?"

No Pre-emptive Considerations

The rationale of the simulation has mostly to do with what future consequences actors take into consideration and what they leave out. Real players or real national actors would reckon differently. Mostly the simulation errs on the side of attributing too-perfect understanding to the actors, but it also leaves out at least one important consideration: the added incentive that comes from forestalling other actors' possible commitments--that is, pre-emptive considerations.[16]

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[13]See page 27 below. [14]See pages 32-33 below.

[15]See page 13 above.

[16]Although chapter ii is all about a minimal model of pre-emption,

When I have tried to construct a detailed algorithm taking concurrent independent steps of decision making into account, the logic gets caught in a circle of mutual second guessing or a stymie of "No, no, after you, my dear Alphonse." That is why, in this model, a simulated actor, when weighing his own possible commitments, takes into account commitments which others have already made and commitments which others might make in response to his own possible commitments, but not commitments which others could be making at the same time, independently of his own choosing.

Objective War Prediction

In the three families of runs principally discussed, simulated actors guess war outcomes accurately.

The pilot model does make allowance for "subjective war efficiency factors," *k*. Using these factors an actor is made to imagine that his side would wreak *k* times as much loss and take *k* times as much booty as they actually would. (He also imagines that the other side is $1/k$ effective; this complementary assumption actually makes the computer program simpler). Neutrals were supposed to be objective, $k = 1$.

A few test runs were made which varied the subjective war efficiency factor. They showed merely that actors who tended to think themselves invulnerable would rush into war and that the opposite kind stayed out unless attacked. I saw nothing of interest in the way it happened, and have set the matter aside.

Objective Prediction of Subsequent Diplomatic Events

The simulation also attributes objective foreknowledge to an

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the complementation of the two chapters and their models is nominal. There is no synthesis of the models or of their results made here.

actor when he predicts the outcome of all future blind bargaining subgames that would ensue if he presently commits himself one way or another. This super-rationality is partly the effect of theoretical perplexity and partly of technical inability. The technical inability was simply that a provision of subjective predictions and their utilities would have overflowed the available memory of MANIAC III. The perplexity is about how an actor's predictions are contaminated by such things as fears and wishful thinking.

One such sort of contamination left out of the pilot model I could now cope with. It may be that actors irrationally discount a further development by giving too much weight to an intermediate diplomatic situation. The following version[17] of history serves as illustration. In July, 1914, the German Foreign Ministry might well have appreciated that England would not overlook an attack through Belgium, but the General Staff rather limited its views to the Schlieffen Plan, which did not engage the bulk of British might. (To make this illustration I want to simplify, discount the British Expeditionary Force, and say that the German General Staff essentially left England out of the reckoning.) Since the Kaiser usually evinced a healthy respect for the British Empire, might one not say that at the last he failed to take England seriously enough? One might say that the German reading of British intentions set too low a probability that England would do as she said, but I think that a truer description would be a contamination of reasonable diplomatic prediction by a more limited view.

The pilot model could readily be modified along these lines.

[17]It is out of my competence to judge its truth as history. See, however, the tearful interview between the wife of the British Prime Minister, the German Ambassador and his wife in The Autobiography of Margot Asquith (London: Penguin Books, 1936), II, 126.

That is, an actor could bid for a possible coalition set as if its value to him were a mixture of war fought out by that set and of war fought out by that coalition set which would actually be formed (the ultimate successor I call it).

I think that modifications like this, but probably further reaching ones than this example, are necessary to change one aspect of pilot model behavior. This aspect is the drawing out of the bargaining among n actors to $n-1$ distinct diplomatic events because only one actor commits himself in each blind bargaining subgame (the victim is committed by his attacker in the first subgame).

The present assumption of objective prediction causes this. When an actor first comes to desire a possible coalition he does not wait for his future partners to bid in the present subgame to join him. Instead he attacks the victim, or joins in, on his own hook, knowing that his partners-to-be will join him in subsequent diplomatic events. This they do--one by one.

Thus the simulation, in drawing out the sequence of diplomatic events, makes a poor use of the blind bargaining subgame. In any one such subgame no alliances are made at all, let alone transfer of possible alliance from coy preferred partners to willing second choices. Such transfers were the whole intended meaning of the blind bargaining subgame.

Either simplification or enrichment would improve the model technically. The presently unused alliance features of the blind bargaining subgame might be discarded as superfluous and possibly misleading. Otherwise, more intricate use, and almost certainly additional structure, are required to bring these features into play.

Balance Keeping Weighed Against Growth

The most elementary hard choice to put before an actor is that between extra growth of his own SIZE and balancing (equalizing the other members of) the system. Of course, a real actor may have private reasons, such as cultural unity, for not wanting to grow beyond a certain fraction of the system. For a first cut, however, one may assume, with Kaplan,[18] that they all desire a margin of superiority if only to ensure their safety against chance[19] redistribution of power. Technological break-throughs and bad harvests are examples of such chances. Anyone's desire for the extra margin beyond the "fair share" will be enough to keep the system in conflict.

To represent this weighting of extra growth vs. balancing, the straightforward decision was to describe each actor's satisfaction with a SIZE distribution as a difference of two terms.

His own SIZE in that distribution is given positive value. From this one subtracts a measure of dissatisfaction with the degree of foreign imbalance.[20] How to measure foreign imbalance is the subject of the next subsection. The dissatisfaction an actor feels with foreign imbalance

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[18]M. Kaplan, "The Systems Approach to International Politics," New Approaches to International Relations (New York: St. Martin's Press, 1968), p. 390.

[19]Elsewhere I have left chance disturbances out of the model. This does not mean that objective disturbances could not be superimposed, still less that actors could not imagine and fear them.

[20]Actually it makes no difference whether one defines balance in terms of the whole system or the foreign part of the system. Since an actor regards his own SIZE as a special case it is natural for him to think first about the balancing of the foreign system. It is also one degree simpler than balancing the whole system.

If an actor, customarily on the lookout to increase his own capabilities, ever does admit that his own wings need to be clipped he can still do so under the domestically palatable justification that being rich he can afford especial sacrifices toward the balancing (defence needs) of others.

is that measure multiplied with his personal coefficient of disutility for foreign imbalance (acronym CDFI).

Obviously this coefficient is interpretable as a measure of an actor's commitment to short-range balance-keeping. The starting point of this project was to check the tautology that the more committed actors are to balance keeping, the stabler the system will be; that is, they will not kill one another.

How to Measure Foreign Imbalance

There are two kinds of question that can be brought under the head of a measure of foreign imbalance.

The first considers reassessing the weight of a given measure in order to take more extreme imbalances more seriously and less extreme imbalances less seriously (or <u>vice versa</u>). It is equivalent to changing the simple linear relationship assumed in the previous section between foreign imbalance and an actor's dissatisfaction with it. It does not consider the relative effect of different actors' (the smallest actor's, the largest actor's, etc.) changing SIZEs.

I have made pencil and paper calculations[21] to see if some (Family A) of the pilot model's performance would be affected by squaring the measure of foreign imbalance. Squaring is one way of giving relatively more emphasis to the more extreme imbalances. After rescaling the CDFI, however, the behavior remained roughly the same. This was only to be expected, since the distributions that come into question (actual and rejected postwar SIZE distributions) are not widely different.

The main question of how to measure foreign imbalance concerns changes in the computation of foreign imbalance which do affect the

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[21] See pages 23-24, below.

relative weight given different actors' (the largest actor's, the smallest actor's, etc.)SIZEs. A change in the method of computation must produce no significant change in the model's behavior or else a theoretically significant interpretation of the difference between the two methods of computation must be found. Otherwise the results are <u>artifacts</u> of the computer technique.

There are too many different functions of <u>n-1</u> variables for one to dream of canvassing them all and considering, then, which might be interpreted as measures of foreign imbalance. I have thought of a few alternate measures of foreign imbalance and so far the pilot model's behavior does not suffer inexplicable changes from the substitutions. I will speak of four here:

a. Largest actor's overweight
b. Smallest actor's underweight (for a good reason this is a bad measure)
c. Total deviation from the mean
d. "Balance the next war" (which was used in the pilot model simulation)

By a foreign actor's <u>overweight</u> (underweight) I mean the amount by which he exceeds (falls short of) the mean SIZE of the foreign system.

<u>Top dog's overweight</u>

The use of the largest actor's overweight as measure should have made the pilot model behavior more obviously tautologous. It seems clear that a concern about the largest actor's overweight will produce Grand Coalitions against him. When the actors are not sufficiently balance-oriented in this sense, the others can be expected to gang up on the smallest actor for the same reason they do in the pilot model: he is the easiest victim.

Bottom dog's underweight

If the actors take the underweight of the smallest actor (his elimination is, after all, the criterion of instability) as their measure of foreign imbalance, then a sufficient concern will, again, be enough to keep them from ganging up on him. They will help him profit in war, but rather at the expense of the second smallest actor than of the largest, again because the smaller is the easier victim.

This pattern of action, further pursued, would have the absurd result that a concern for foreign imbalance lets the largest actor roll up the system.

The fallacy, however, is readily recognized. One doesn't leave the largest actor out of consideration. The classical writings, e.g. Hume's Essay XXIX,[22] habitually speak of balance as if it means especially restraint of the preponderant actor.

Total deviation from the mean

If one doesn't give any special consideration to the largest actor's overweight, the simplest foreign imbalance measure, taking all (foreign) actors' SIZEs into account, is the total (absolute) deviation from the mean SIZE, that is, the sum of all overweights and underweights.

Again, pencil and paper calculations were made to compare the effect of substituting this measure. The shape of the results (of Family A) remains the same; some boundary values are changed slightly[23]

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[22]D. Hume, Essays, Literary, Moral, and Political (London: Ward, lock, and Tyler, n.d.), p.201.

[23]It will be told below, p.33, how the durability ratio must be pushed up from 67 per cent to 84 per cent to make three-two wars possible in Family A. Using the total-variation-from-the-mean measure, the durability ratio must be pushed up to $87\frac{1}{2}$ per cent for that to happen.

(the same can be promised for Families B and C).

Balance the next war

The rationale of this measure is that an actor projects discrepancies between the sides of future wars as the kind of imbalance he is going to find it necessary to overcome.

Partly it was the metaphor of the balance between two armed camps as a scales and the figure of a ship to be trimmed that directed attention to wars between equal numbers of foreign actors and away from the Grand Coalitions which actually occurred in the model. It is more natural for an actor to think of reversing the balance when the others are already nearly evenly matched.

With five actors four foreigners, A, B, C, and D, can fall into opposing pairs three different ways: AB vs. CD: AC vs. BD; and AD vs. BC. For measuring foreign imbalance in the pilot model, the absolute differences in these three wars were added. Using the same letters for the actors' SIZEs, the formula for foreign imbalance is, thus:

(Eq.2) Foreign Imbalance $= |A+B-C-D| + |A+C-B-D| + |A+D-B-C|$

This rationale seems to lean toward the assumption of more evenly divided wars. When the pilot model, with this assumption written in, repeatedly gave uneven wars of four actors against one instead of relatively even wars of three against two, one of the things I did was to compare this measure more thoroughly with the others.

Foreign imbalance computed under the "balance the next war" scheme proves to be just either four times the top dog's overweight or four times the bottom dog's underweight, whichever is worse(greater).

Comparison with the total variation measure is more complicated. If the total variation measure is held constant, at one unit say, the "balance the next war" measure ranges from one to two. Lesser values occur with two nearly equal top dogs and two nearly equal underdogs. Greater values occur when one of the four actors is particularly over- or underweight and the other three much of a common size. That is, the "balance the next war" measure gives proportionately greater weight to these latter kinds of imbalance than does the total variation measure.

The full range of variation between these two measures did not affect the pilot runs because, again, not very widely different hypothetical distributions ever came into question there. A later experiment might behave with greater variety. One can only hope that its more varied conditions would also give a theoretically meaningful handle to the differences which changed the realization-model's behavior.

Complaisance Factor

The name "complaisance factor" was chosen because I expected it to be the parameter which would describe the transfer of alliance hopes from preferred but reluctant partners to willing second choices. Since alliance formation within the individual blind bargaining subgame has not materialized,[24] this factor merely describes the speed with which an actor takes a unilateral decision.

The complaisance factor is defined as the constant of proportionality between the marginal utility which a biddable coalition set seems[25] to offer and the time before the deadline at which the actor bids for it.

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[24]Because an actor knows which partners will join him, see pp.18-19.

[25]The logic of what a coalition set seems to offer has been stated in the previous subsections. An actor predicts the course of future diplomatic events, that is to say, the coalition set which will be the ultimate successor of the one he bids for now. He predicts the outcome of war fought by the presently formed (not the biddable)coalition set,

Projected effect on alliances made within a blind bargaining subgame

In a blind bargaining subgame which reflects a competitive situation, a very complaisant actor, that is, one who quickly makes all his bids, is likely to get alliances, but not very profitable ones. He may have climbed down to his fifth choice, which is somebody else's first choice, before a potential partner will be ready to agree on what is second choice for both. Conversely, an intransigent actor who holds out for his first choice alliance, will often get no allies. Both extremes are, thus, unprofitable and maximization of utility indicates that an actor should choose a complaisance factor comparable with those of actors in similar strategic situations.

For a given range of blind bargaining payoffs, such as those that might arise in a war game model, it would be interesting to note whether there was an advantage to an actor in having a complaisance factor more or less than average. Strategic inputs, such as a comparatively ruinous cost of having no allies, should, realistically, affect the tempo of diplomacy. A bias in favor of above average complaisance factors would correspond to a diplomatic speedup and the reverse bias to aloofness.

To make such inquiries with the pilot model, however, it must be emended so that alliance are struck within a blind bargaining subgame.

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which will happen if bargaining goes to the deadline.

The difference between the satisfactions felt with these two states of affairs may be taken as the relevant marginal utility. An unused feature designed for the pilot model, and discussed in the next subsection, imports a difference to the distinction between "satisfaction" and "utility." On what has been stated so far, it must seem empty precision to insist on the following. Not postwar SIZE distributions but transitions from present distributions to postwar are the objects actually bid for. Therefore, the term "utility" should be reserved for transitions and another, "satisfaction," found for the end points of those transitions.

Effect on alliances made over a sequence of subgames

It was expected that varying the complaisance factor could make a difference in behavior, even without changing alliances formed in a subgame. The actor who commits himself first may be able to force others to a somewhat less preferred course. For example, this might happen. A attacks C, apparently putting his own neck on the line, but he knows that B and D will help him. D and E would rather attack B, but the rules permit them only to join one side or the other of the A versus C war. Making it possible for D and E to fight B separately would not answer all possible problems under this head.

Here is an actual example from Family B. In the typical critical situation of those runs the deviant is the second largest actor, and as such,[26] commits himself before the other middling actors do. This being so, the largest actor may start a war on the smallest, knowing that the deviant, and then the others, will join him. The largest actor, like all the others, wants a short-run SIZE profit, but he cannot ordinarily find a more acceptable victim--himself. The deviant chooses the wrong side because he values the slightly greater SIZE profit. The other actors join the badwagon, because a three against two war is quite unprofitable and they want profit.

With increased common non-deviants' CDFI, the largest actor is slower[27] to want the unbalancing raid on the smallest actor, but the smaller three actors are still eager for a war against him. The deviant

[26]Because time of bid is proportional to absolute satisfaction, larger actors, who have more SIZE profit to make, bid first.

[27]He only compares with the status quo. He doesn't realize that alliance with the deviant is his only chance to stave off a Grand Coalition, cf. subsection "No Pre-emptive Considerations," p.16 above.

cannot force his preferred issue by attacking the smallest actor; it would give the largest actor a chance to get on the right side.

These are the pilot model reasons I see for the facts that (i) there is still Type Two (dynamically stable) behavior in Family B but that (ii) the lower[28] CDFI boundary value of that type is raised. In other words, Type Two surrenders some runs to Type One in the passage from Family A to Family B (deviation of one actor).

The sequence does not affect Family A essentially.[29] Middling rank actors all prefer the same thing, a coalition against either the largest or the smallest actor. Of these two actors, the victim has no options; an attack on someone else will bring him no allies. The non-victim is quite glad to join the middlers' coalition. Thus it does not matter if any actor expresses his preferences quickly or slowly.

Quantifying Utilities

The pattern from a "Dutch Auction" or a blind bargaining game seems a useful way to conceive the quantification of utilities. An object is rated highly if an actor bids for it early. Another object is rated less highly if one waits till he is near getting a booby prize

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[28]The higher upper boundary, that is, the transfer of runs to Type Two from Type Three(peace), may be due to this. The deviant, not caring for the balance, was willing to disturb the initial perfect balance for a quick profit. Once he declared on an actor, that balance was spoiled, so others joined in. Had they cared still more for balance, they would have left him and his victim to get poorer together, so, of course, he would not have attacked.

This all depended on a lucky chance in the pilot model, but a reasonable one. Namely, the deviant was able to find a case where the others would choose to come in on his side, not on that of the actor whom he attacked.

[29]I dismiss from consideration any conceivable runs at the borderline CDFI where middling actors, because of their relative rank, might incline to different types of behavior. Assume--it is only a slight smoothing of detail--that they all change types at exactly, rather than approximately, the same point.

before taking it as second best. This is using operationally ascertainable behavior for the quantification.

In the von Neumann-Morgenstern foundation of the theory of utilities[30] there is a wholly imagined subjective lottery with two prizes, grand and booby. To publish his utilities an actor must fancy a lottery where the subjective probabilities of the outcomes make him indifferent between this lottery and the object whose utility is in question, Though indifference is operationally ascertainable, the imagined lottery and its subjective probabilities are not. They can be published only by assertion.

My pattern would have an actor publish utilities more exclusively through deeds. The time at which he grabs for the second prize is an objective indicator about how he feels about it as compared with his increasing chance of missing the grand prize.

Logical Difficulties

Is it necessary to say that an actor's expectations decline uniformly with time? May not they drop in sudden breaks or at least decline in other functions than the linear?

I believe that much of the seeming difficulty can be conjured away by finding the right viewpoint; mathematical theories are useful for such finding. It seems significant that the von Neumann-Morgenstern formulation has to be disentangled from the problem of distaste for risk.[31] Some of these problems can surely be dealt with analogously.

Perhaps it is convenient that the distaste for risk can be tacked on to utilities already associated with timed bids. Then a more

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[30] R. D. Luce and H. Raiffa, Games and Decisions (New York: John Wiley and Sons, 1957), pp.21ff.

[31] Ibid.

perspicuous analysis of that risk can be given than when the utilities themselves are based on risky alternatives.

Distaste for Risk in the Pilot Model

These considerations were designed into the pilot model but not used. Actors could attach a different marginal utility to increases in satisfaction than to decreases in satisfaction. Thus a round trip to prosperity and back or--here distaste for risk comes in--an even gamble between equal gain and loss might be felt by an actor to contain unequally offsetting rewards and punishment. Take a national actor as a government responsive to the needs of a dominant social system and examples can be imagined either way.

A Venetian plutocracy might suffer worse from a contraction in trade than it gains from a corresponding expansion: marginal merchants are hurt badly and make trouble for the government. A military aristocracy might suffer less from defeat than it gains from corresponding conquests: superfluous younger sons are killed in a defeat.

Short Range and Long Range

The simulated actors carry out their prediction of consequences so far as to predict postwar SIZE distributions. This is the necessary minimum range of prediction, for that is the range of the commitment; after the war, an actor can commit himself again.

Kaplan has asked me to deal with the possible objection that this rationale of taking the minimum period of prediction amounts to taking a very short-run view. This objection implies a just criticism of the pilot model as so far developed. The model contains only a rudimentary projection into the next round (the "balance the next war" measure for foreign imbalance). Neither does interpretation from a longer range point of view add much, at least to Family A of runs and results based on them. In this subsection I shall discuss what interpretation and style construction can do to remove this objection about a short sighted model.

In principle, the pilot model does not contain an unreasonable and incurable short-run bias. First of all, the minimum prediction to postwar SIZE distribution is the natural one. An actor would naturally express his predictions about the hypothetical longer range in terms of the evaluation he places on an immediate postwar SIZE distribution, that is, he would modify his short-range style. For example, if he thinks that a certain foreigner is especially likely to cause trouble, his response will boil down to placing a premium on present reduction of that actor's SIZE.[32]

The radical alternative to the apparently short-run nature of the pilot model rationale would be to have actors plan future choices in advance. There is a theorem of Harold W. Kuhn which says that this adds nothing to making each choice as it comes, provided one can remember the reasons for the previous choices.[33] This justifies, I believe, the construction of forward looking strategies in terms of modified style functions. (These are defined as giving an actor's "satisfaction" with immediate postwar SIZE distributions.)

Finally, interpretation of the pilot model from a long range point of view is straightforward, if not always rewarding. Since the

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[32]One contemplated refinement of the pilot model is the counter-deviant style, wherein an actor scores the destabilizing behavior of others and marks offender(s) for especial reduction. Although, strictly speaking, no projection into a long future appears on the surface, the forward looking rationale in the text obviously belongs to such a style.

[33]See the topic of behavioral strategy, e.g. Luce and Raiffa, pp.159ff.

styles of play defined in terms of maximizing postwar "satisfaction" are not intrinsically short-range, nothing prevents evaluating them from a long-range point of view.

To make this reinterpretation one does not stop at describing actors' styles as hegemonial or balance-oriented from the short-run point of view. Instead one takes styles, however obtained, and tests them in the model[34] against one another. Then one can ask whether hegemonially or balance-inclined actors would choose such styles as strategies for a longer play of the model.

Applying this to Family B. one sees that the deviant tactic is an incomplete recipe for hegemony. The deviant gets rid of others, but only as the jackal of a larger actor. If this is the best he can do, the jackal will be eaten last. On the other hand, deviance of this sort does begin to clear the competitors away and stochastic luck added to the model could give a jackal a hegemonial chance.

For balance oriented actors, it emerges that raising their commitment to the balance (CDFI) will overcome one or even two deviants. Against a magority of deviants, however, there can be no resource.

Suppose, however, that there is some resistance, perhaps domestic, to raising the CDFI. Then it follows that balance commitment entails a price. If balance keeping fails, it is non-deviants who first suffer.

Results Which Are Not Sensitive to the Bargaining Sequence

Of the two points here, the result which most interested

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[34]This may be the first useful point at which to add some effects, such as stochastic SIZE shakeups.

Kaplan[35] and me was the continual occurrence of grand coalitions. It contravened assumptions made in the general literature,[36] and, as we thought,built into the pilot model through the "balance the next war" rationale.

The reason for this to be found in the pilot model is the <u>durability ratio</u>. It has to be pushed above two-thirds before three-two wars become profitable and above sixteen-nineteenths before a common CDFI can be found which will make actors willing to disturb the perfect initial balance for a mild profit from a three-two war, while still rejecting the severer upset of the balance occasioned by four-one wars. In other words, upon raising the durability ratio past 84 per cent, Type Two of Family A subdivides into a type of highly balanced three-two wars as well as the former four-one wars on the largest actor. Looking at the removal of boundary value CDFI, one must say that this is at the expense of runs transferred from Type Three (peace).

Dynamic Stability

Parenthetically, it is the Type Two runs which, though for rather formal reasons, give the first results of any theoretical interest. (The truism about increasing CDFI making for increasing stability was just that, and cannot be called a result though if its falsification had occurred, that would have been something to look into.)

Type Two runs, however, were not necessarily to be expected from the start. If the model had yielded only Types One and Three, that is, "war to annihilation" and what is inaction when starting from initial equality, it still might have been fitted into a theory about

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[35]Kaplan, "The Systems Approach . . . ," p.386.

[36]See note 4, page 5 above.

dynamically stable systems, that is systems which fight but not so as to annihilate an actor. Stochastic SIZE disturbances would make Type Three fight. For high CDFI and unequal initial SIZEs, actors would fight against the currently largest until approximate equality was attained. Under the circumstances the model would still have looked useful as a representation of balancing action which offsets stochastic increases in capabilities.

The dynamic stability in Type Two is more interesting because it is action caused by the behavioral styles and expectations of the actors as represented in the model, and not by external causes.

This behavior also seems to reproduce an important nuance of the classic "balance of power" idea, a certain oxymoron: systems are expected to achieve long-run stability by short-run action that comes close to being destabilizing. These Type Two actors desire a short-run gain greatly enough to spoil an initial perfect balance. The substitution as victim of the largest actor for the smallest actor costs the middling actors very little short-run growth. Thus it is possible to find a viewpoint--necessarily not the best viewpoint--from which these actors seem largely inclined to instability, and yet--the eighteenth century might speak of an "invisible hand" or the balance of an international machine--their interaction is so arranged that they achieve stability. Of course, this is because they consistently choose the largest actor as victim, and it takes no computer model to see the futile stability of that habit, but in this respect the "balance of power" idea is simple.

Remarks on Riker

William H. Riker rests a picture of evenly matched wars ("minimum winning coalitions") on a principle for which he gives mathematical argument:[37]

> The Size Principle: In *n*-person zero-sum games, where side payments are permitted, where players are rational, and where they have perfect information, only minimum winning coalitions can occur.[38]

I have found my realization-model useful for following Riker's argument.

My first objection is that his principle, even if true, may be misleading. Side payments might conceal a result which robs the Size Principle of meaning: a bribe to keep an actor from going over to the minimal winning side is rather like tribute to a conqueror.

In my opinion Riker should never have brought the notion of winning into game theory. He defines a coalition as winning when it gets a positive payoff. This, however, is not a strategically invariant concept as the following argument proves.

In any game, suppose an extra actor who levies a uniform tax on the other actors. He will join no coalitions and engage in no side payments because he is strategically irrelevant. Although he does not change the game-theoretic strategic situation of the original set of actors, his levy (or his contribution) changes the "winning" and "non-winning" coalitions.

Since the minimum winning coalition is not an invariant concept under game-theoretic equivalences, one expects a flaw in Riker's argument for it. It comes at the last step of his derivation, which assumes

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[37] Cf. W. Riker, The Theory of Political Coalitions (New Haven: Yale University Press, ~~1962), pp.247-278, with Luce and~~ Raiffa, pp. 180ff., especially at p.186.

[38] Riker, The Theory . . ., p. 32.

that games where additional members add something to the winning coalition's payoff are "quite rare."[39] He evidently confuses the payoff function's peak, which is an invariant, with the non-invariant point where it becomes positive. (The function he is graphing is coalition payoff against additional members. I do not complain that members could be added in different orders; it is a useful mathematical imprecision.)

The war equations in my model,[40] however, are a counter example to his assumption. Extra actors in a winning coalition add not merely to a coalition's payoff but also to each member's share. (This latter is necessary to defeat the real argument for the minimum winning coalition, which is that extra members are not a profitable proposition.) The reason these equations give every winner more is that the extra member bears some of the losses inflicted by the victim but takes his own share of the booty for himself.

Finally, there is another way in which minimum winning coalitions are misleading. They are not the same thing as the military winning side. Suppose three actors out of five come into a coalition for their best result. This may be to attack one of the others and suffer the fifth to join them. They may not care to fight off his unwelcome assistance.

Technical Details

This section contains a flow chart, Figure 2, a list of variables and concepts in the pilot model, and statements of extra formulae and procedures useful for anyone who wants to reconstruct the pilot

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[39] Those which "v(S) (the payoff function to coalition S) has in part a positive slope," ibid, p.278.

[40] See (Eqs.1), p. 9, above.

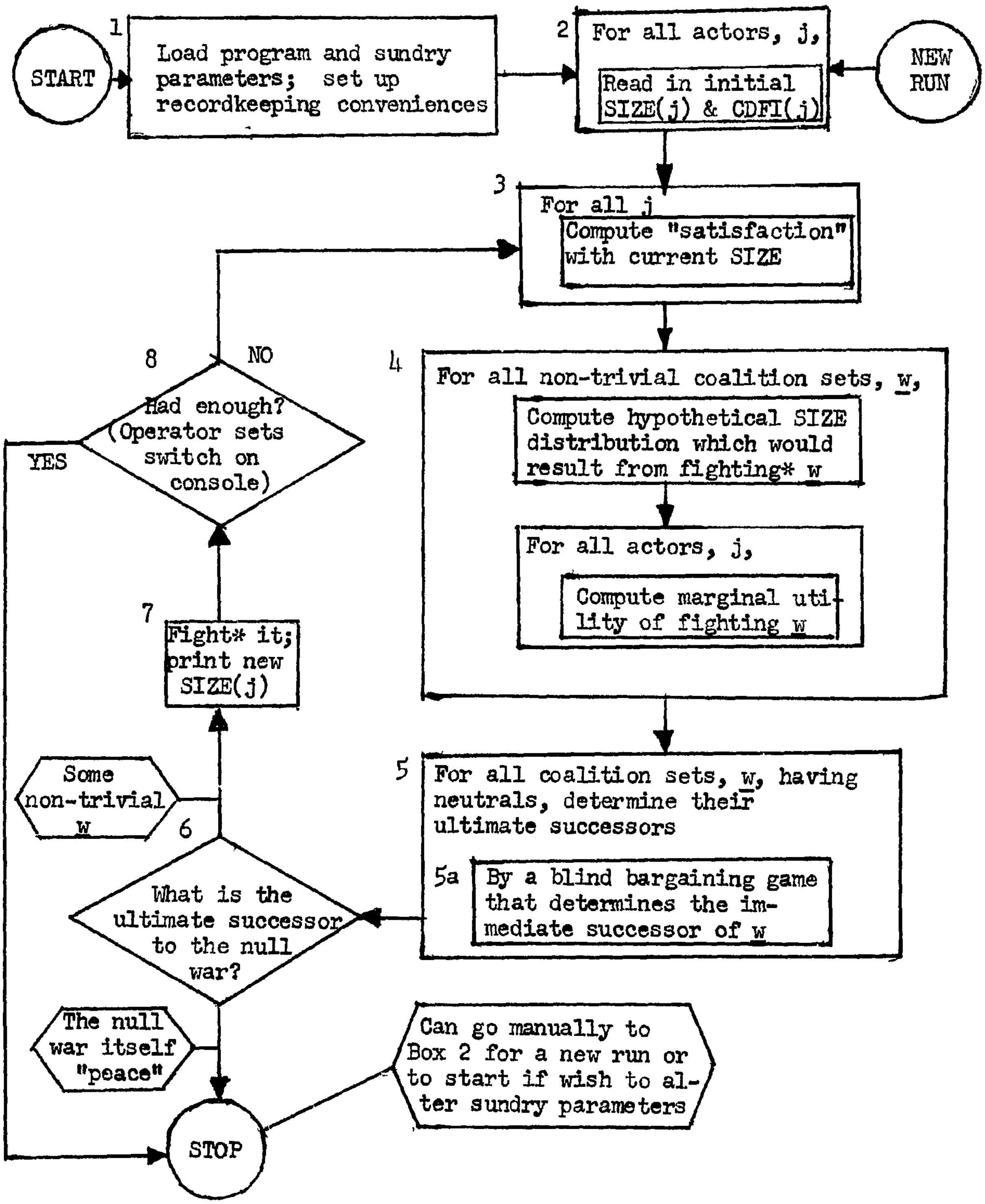

Fig.2--Pilot Model Flow Chart

model simulation. The previous section is prerequisite. It motivates and explains.

TABLE 1

LIST OF VARIABLES AND CONCEPTS IN THE PILOT MODEL

Symbol	Name	Comments
n	The number of actors	This parameter was given the value 5
j	An integer index distinguishing actors	Ranges from 1 to n
SIZE(j)	An actor's size	Initial values are input
FORIMB(j)	Foreign Imbalance with respect to j	
SATIS(j)	"Satisfaction" to actor j of a distribution of SIZEs among members of the system	
CDFI(j)	Actor j's coefficient of distaste for foreign imbalance	A parameter
w, w'	Coalition sets or diplomatic situations	Partitions of the set of actors into opposed sides--"Red" and "Blue" and neutrals. For n=5 there are 91 such, e.g. AC versus DE (B being neutral). Coalition sets are considered sometimes as proceeding immediately to war, sometimes as being modified to other coalition sets by the commitment of neutrals.
w = 0	The null war	All actors neutral
SIZE(j;w) FORIMB(j;w) SATIS(j;w)	j's Size, Foreign Imbalance, and Satisfaction as they would be after fighting out the war, w.	

TABLE 1--Continued

Symbol	Name	Comments
$V_j(w)$	Immediate value (marginal utility) to j of fighting out war w	
(w')	(Potential) Immediate successor, w' (of w)	A coalition set, w', which can be created from w by a single diplomatic event.
E(w';w)	Effective set of actors for forming w' from w	
D(w';w)	Effective date at which w' is obtainable from w	Largest amount of time before a bargaining deadline at which all members of the effective set, E(w';w), are amenable to the diplomatic event of w' becoming the immediate successor of w
	Ultimate successor (of w)	Coalition set formed by the whole sequence of diplomatic events which ensue upon the condition described by w. Hence the war to which w will lead.
$V_j*(w)$	Ultimate value to j of having diplomatic commitments described by w	Hence the immediate value of w's ultimate successor.

Formulae Used in Boxes 3 and 4 for Preparing
Vàlues Used in the Procedure of Box 5

For convenience, denote the SIZEs of the four other actors than j as A, C, D, E. Then the model's mode of computing FORIMB(j) is a sum over the three ways that foreigners can be divided into opposing pairs. Each term is the absolute difference between the sides so opposed.

(Eq.2) $\text{FORIMB}(j) = |A+C-D-E| + |A+D-C-E| + |A+E-C-D|$

FORIMB(j;w) is computed by the same formula, only with hypothetical postwar w SIZEs.

(Eq.3) SATIS(j) = SIZE(j) - CDFI(j) x FORIMB(j) (Box 3 of Fig.2)

(Eq.3') SATIS(j;w) = SIZE(j;w) - CDFI(j;w) x FORIMB(j;w) (lower box)

(Eq.4) $V_j(w)$ = SATIS(j;w) - SATIS(j) (in Box 4 of Fig.2)

Procedure of Considering Coalition Sets, w,
to Determine their Ultimate Successors. (Box 5)

1. By initial presuppositions (recordkeeping conveniences of Box 1): coalition sets where all actors are committed are their own ultimate successors. Other w are to be scanned in an order taking first all those with one actor neutral, then those w with 2 actors neutral, etc., the last w considered being w = 0.

2. For each such w, choose its immediate successor w' by the subprocedure (below).

3. Now determine that the ultimate successor of w is the ultimate successor of w'; item that the ultimate value of w is the ultimate value of w':

$$V_j^*(w) := V_j^*(w')$$

The important thing to note about this is merely that the list of w must be scanned in an order as prescribed so $V_j^*(w)$ will be known for use in (Eq.5), p.41 below

Blind Bargaining Game
Subprocedure for Choosing an Immediate Successor
to any Coalition Set w Where There Are
Uncommitted Actors

1. Two cases, w = 0 or w ≠ 0, affect the range of potential immediate successors and their corresponding effective sets:

> If w = 0, every other coalition set, w' is a potential successor and must, in principle, be considered twice over, once with the red side as E(w';0), once with the blue side as E(w';0), since either side may be the attackers.

If $w \neq 0$, then consider as potential successors those $\underline{w}'$ which can be formed by the adhesion of $E(w';w)$ (some or all of the neutrals from $\underline{w}$) to one side or the other side in $\underline{w}$.

2a. In addition, one also permits the "non-event" that $\underline{w}$ might be its own successor. This possibility has effective date $D(w,w) = 0$.

2b. The effective date for obtaining a potential immediate successor, $\underline{w}'$ from $\underline{w}$, by force of the effective set, $E(w';w)$ is:

$$\text{(Eq.5)} \qquad D(w';w) = \underset{j \in E(w';w)}{\text{MINIMUM}} \left\{ V_j^{*}(w') - V_j(w) \right\}$$

3. Of all potential immediate successors to $\underline{w}$, the chosen immediate successor is that $\underline{w}'$ which maximizes $D(w';w)$. In the case that $w' = w$, it is also its own ultimate successor.

Equilibrium Points Taken from the Pilot Model

Pencil and paper calculations are useful when one assumes certain steady states of the model. In fact, they make it quite possible to dispense with a computer for many purposes. This is quite commendable, in that the computer is only supposed to be a temporary aid in finding interesting theoretical relationships. It is also likely when, as here, major complexities of the realization-model have not been put to work.

There are three relative SIZE distributions of five actors corresponding to three types of behavior. Each is a geometric sequence, and war (or peace) of the corresponding type preserves the relative SIZE distribution, though rearranging the actors. Again I assume booty and loss factors of 10 and 15 per cent. These are the three equilibrium distributions:

For peace:	1.0,	1.0,	1.0,	1.0,	1.0
For 3-2 wars:	1.046,	1.023,	1.0,	.978,	.956
For 4-1 wars:	1.225,	1.107,	1.0,	.903,	.816

The three families of pilot model runs concern principally departure from the first equilibrium point to the other two. One could divide the issue into two parts: CDFI low enough to permit departure from the first calm, and the higher CDFI which are still consistent with action in one of the other equilibrium points.

Many of the assertions made about the computer model in this chapter are partly based on such calculations.

CHAPTER II

A MINIMAL MODEL

This model, originally a simplification of the pilot model, was addressed to the problem of the optimal number of actors in a "balance of power" system--not, of course, what the number is--but reasons for fancying such a number. Being simple, the model's behavior is totally known and presents the most elementary considerations. In the model's development several relevant ideas of Kaplan and Burns were clarified but not incorporated.

Description

This realization-model for the "balance of power" has as little detail as is consistent with usefulness. It comprises a variable number, n, of actors existing through continuous time, t. What happens in this model is that n can be reduced 1 at a time, though not below 1. These reductions are either pre-emptive and immediate or non-pre-emptive and slower. The model turns on A_n, which is the normalized discounted life expectancy of the average member of an n-actor system. Pre-emption[1] is assumed to occur in a system whensoever the background risk (of non-pre-emptive reduction) makes life shorter than it would be in the reduced system.

Pre-emption or instant reduction may be interpreted as some $n-1$ finding themselves agreed that life would be longer in the system without

[1] In this whole chapter, pre-emption refers merely to annihilation, not to gaining a partial advantage. This is an all or nothing model.

the other actor. The background risk of non-deliberate pre-emption may be interpreted as arising from various causes outside a normal security-oriented rationale. For one thing, internal accidents, or blunders by other actors, may make it too difficult to maintain an actor's Great Power status. For another, actors may occasionally deviate hegemonially and come to regard the solitariness of survivorship as more splendid than mere existence.[2]

Two parameters are implied for the model. The first is M, a Discount Rate. It represents the degree to which a future event, such as a year of life, matters less to an actor merely because it is in the future. The second is P, a Background Risk Rate. The way that reduction should depend on this rate is the primary door for bringing more details into the model. The natural critical assumption to make is that the risk goes up as the number of actors. If it went up any faster there would be pre-emption in some large system.

The heart of the model is the recursive expression which gives the value for average membership in the n-actor system, on the assumption of non-pre-emptivity:

$$(\text{Eq.}6) \qquad \text{"AifNO"} = \frac{1 + \frac{P}{M}(n-1)\, A_{n-1}}{1 + n\frac{P}{M}}$$

The exponential rate of discount assumed is M. The exponential rate of background risk assumed is nP. The presence of $n-1$ in the numerator accounts for the 1 chance in n of not getting membership in the $n-1$-actor system.

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[2] See E. Canetti, Crowds and Power (New York: Viking Press, 1963), in the chapter entitled "The Myth of the Survivor" for a readable psychoanalytic portrait of the hegemonial tyrant as paranoid.

The presence of two terms, one weighted by a ratio P/M in each of the numerator and denominator, represents a mixture between the two basic possibilities. When P/M is small, the expression approaches 1, the value of unperturbed existence. When P/M is large, the first terms become comparatively insignificant and the expression approaches the (obvious) formula for instant reduction:

$$\text{(Eq.7)} \qquad \text{"AifYES"} = \frac{n-1}{n} A_{n-1}$$

The variation with the ratio P/M is quite reasonable. The ratio is a relative background risk rate and measures the number and subjective impact of the future chances of reduction upon an actor. If he sees a lot of such chances it is as if the future were quiet. The discount factor (rate) may make him long or short sighted about a given objective incidence of future risks.

A detailed derivation of (Eq.6) will not be given in this chapter. All the interesting points of the derivation are covered by the subsection "Technical Progress," page 54 and note 29, page 66 below.

Behavior and Four Results

Four inferences may be drawn from the simple and completely known behavior of the minimal model. The pattern of this behavior is, firstly, that, for fixed parameter values, smaller systems are pre-emptive and larger systems non-pre-emptive. Secondly, the critical size or boundary between pre-emptive and non-pre-emptive states increases directly with background risk and inversely with discount rate. In fact:

if $1 < n < \frac{P}{M} + 2,$ the system is pre-emptive;

if $n \geq \frac{P}{M} + 2,$ the system is non-pre-emptive.

No Alternation

The first result is that the simplest system is not alternately pre-emptive and non-pre-emptive with varying numbers. Alternation could have been expected on the following grounds. If one size system, e.g. the one-actor system, is very safe, the next size system should be unsafe because actors pre-empt to get into the safer system. Conversely, if a system is pre-emptive and dangerous, e.g. the two-actor system, actors in the next size system should non-pre-emptively avoid reduction to that system. This argument tends to make the three-actor system stable. The expectation of alternation was that under some reasonable unspecified assumptions this alternation could go on to higher n.

That Long Range View Can Cause Hegemonial Grab

Of course, these unspecified assumptions would have had to include something to the effect that the background risk is low, for if the risks of non-pre-emption are great, three or even more actors will scramble for the chance of sole survivorship. It is less obvious at first that the discount rate should be high but the model's behavior does depend merely on the ratio of background risk and discount rates. In effect, actors who discount the future slowly add up a long vista of risks. Despairing that the plural system could last into a distant future, they make a grab for the chance of sole survivorship.

Only One Chance in n of Successful Hegemonial Grab

The chance of surviving an all-out scramble depends inversely on the number of actors. This is the simpleststrategic reason that additional numbers make for non-pre-emption. I call such safety in numbers <u>dilution</u>. Other forms of dilution can be progressively distinguished , starting from this very simple one.

Only One Chance in n of Being Stung for Not Having Pre-empted

Adding more numbers continues to make the system safer for the average actor, but this non-pre-emptive comparative stability hangs by an ever slenderer thread. Surely one actor cannot care very much whether he has nineteen or twenty fellows?[3] On the logic of the minimal model this thread never quite snaps. There is, in fact, a secondary dilution which offsets the bare alternation argument (pre-empt to get to the next smaller system which is still large enough for safety). It is this. If the typical actor[4] gambles on non-pre-emption and loses, there is only one chance in _n_ that he personally pays the price. Thus, once one is clear of the scramble to be king-on-the-mountain, the whole business of pre-emption matters less with more numbers and that is a good, simple reason why actors care so little about the difference between nineteen and twenty fellows.

Comparison with the Pilot Model

Since there is no SIZE differentiation in the minimal model, the three different equilibrium points of the pilot model (peace, three-two wars, and four-one wars against the largest) are lumped into one corresponding state: non-pre-emptivity. Wars to annihilation (Type One) correspond to pre-emptivity. The minimal model, however, is based on the imperfection of equilibrium states; reduction is just slower in them.

The minimal model also paints a much less detailed picture in

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[3]Kaplan, Burns, and Quandt, "Theoretical Analysis . . . ," p.245.

[4]In a more complex model, where an atypical actor goes out of his way not to pre-empt, his chances of paying the price could go up to as high as one in _n-1_.

time of the co-operation between the actors. There are neither diplomatic events of several actors' commitment nor individuals' privately dated bids towards such events.

Members are either cast into outer darkness by their fellows or retain full membership in the system. In this model, there is nothing in between.

Two things, however, the minimal model has which the pilot model so far lacks. The first is a rudimentary consideration of pre-emption. Certainly it is not a detailed picture of one actor edging towards a new commitment, motivated by the fear that while the clock ticks on toward fixing the present commitments, the other actors may be plotting against him.[5] One can only recognize this model's feature as pre-emption in that it makes life expectancies even shorter than the triggering condition makes them.

The other thing the minimal model has is explicit long range motivation. The actors try to maximize (discounted) life expectancy not a "satisfaction" descriptive of short-run behavioral strategy (tactics).

These are the formal differences between the pilot model and the minimal model. There is also a difference in the purposes of their construction. The pilot model was built to solve a technical difficulty about selecting a coalition from a plethora of choices and then used in a fishing expedition for theoretical results. The minimal model was designed in an attempt to hunt down and define certain specific ideas about the "balance of power" model. Unlike elephant guns and fishing rods, one uses the simplest possible tool when a goal is in mind and one putters with more complicated devices.

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[5] See pages 16-17 above.

There is probably a useful synthesis of these two models which has not yet been built.

Background and Development

The results make some sense without mathematics. It may be mostly idiosyncratic that they appeared after mathematical contortions, digging up the most obvious last. I think, though, that it is a property of mathematical modeling to take one an even longer way round to truisms than verbal theorizings take, and they sometimes take quite a long circuit.

This section marks the stages by which the minimal model emerged. It was a two-fold simplification. Half of the job was peeling away (sometimes for later reconsideration) features of the first calculations. The other half was recognizing the shape of theoretical arguments in the literature and seeing that they were not in the model. At the end, Kaplan's and Burns' ideas had been distinguished from one another and from the ideas of the minimal model.

It seems fair to say that Kaplan and Burns did not see the ideas which are expressed as results of the minimal model as independent ideas for the "balance of power" capable of separate treatment. The problem of finding these ideas in other people's writing is not their striking novelty but that these simplicities are naturally confused by complications and refinements when one talks about the real "balance of power" idea. It takes a mathematical model to confine one to such simple ideas. Distinguishing confined ideas, however, also gives a handle on other ideas, in some cases notions for their mathematical representation.

In discussing the pilot model, Mr. Kaplan wished the model were developed[6] enough to deal with an important elementary topic. He and

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[6]Actually it was appropriate first to throw out virtually the

Burns evidently entertained different premises concerning the optimal number of actors for stability in a "balance of power" model.[7] It would take, he said, something like a firm mathematical model to pin down and define the points at issue.

The pilot model seemed inapplicable to this problem because of the actors' (tactical) CDFI. There appeared to be no basis for comparing systems with different numbers of actors and saying what constituted the "same" CDFI. Without such a basis of comparing numbers one could hardly consider the best number.[8]

Prototype Model

I began by writing equations for recursive calculations about the discounted life expectancies of "strong" and "weak" actors. These equations[9] implied a model where, of _n_ actors, one is _weak_, the rest are _strong_. This model operates in rounds of discrete time. Each round, the position of weak actor is taken by one of the previously strong actors. According as the operation of the model is _stable_ or _unstable_ the weak actor survives as a strong actor or is eliminated.

This model, though not novel, seems to be borrowed from the

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entire works of the pilot model. Suppression of detail is very helpful.

[7]Cf. Kaplan, Burns, and Quandt, "Theoretical Analysis . . . ," p.245a. Kaplan's explanations and my reconstructions have ranged over more ground than this. Only after considering other problems did I explore Burns' ideas on how additional numbers destabilize a system through complexity and uncertainty of decision making, cf. Burns, "From Balance . . . ,"in Rosenau, _Reader . . ._, p.355a

[8]As often happens, the obstacle has been removed, but not out of sight. After replacing an actor's tactical CDFI with a long range strategic calculation, there still remains the problem of comparing levels of background risk for different numbers--almost it would seem the entire problem of what size system is stablest. This form of the problem,however, offered some handholds. Before that a few things can be done tentatively.

[9]See (Eqs.8) below, p.51.

pilot model, and more from that model's behavior than from its original (but not fully used) design. This is the derivation. The SIZEs of all actors but the smallest are not distinguished. He, of course, is the weak. Any series of rounds forming a war to annihilation is telescoped into one round. A slightly peculiar feature of this model is that in a stable round the weak actor is always promoted. It is evidently borrowed from the pilot model's Type Two behavior; a real "Sick Man of Europe" can linger for various periods.

The equations of this model (at the end of the next paragraph) are recursive in two senses. The sense which corresponds to their motivation is a recursion in time. The equations give values (life expectancies) for one round with reference to what life expectancies may be in the next round.

Namely, actors score one point for surviving the present round (if they do) plus the expectations of their position in the next round multiplied by F, a discount factor. Another parameter, P_n, is needed to denote the probability of instability in an n-actor system. Denote the life expectancies of weak and strong members of an n-actor system as W_n and S_n, respectively. Then,

(Eqs.8)

$$\text{for } n>1: \quad W_n = (1-P_n)\left[1 + F(S_n)\right] \quad ; \text{ and}$$

$$S_n = 1 + F\left\{P_n\left[\frac{(n-2)S_{n-1} + W_{n-1}}{n-1}\right] + (1-P_n)\left[\frac{(n-2)S_n+W_n}{n-1}\right]\right\}$$

The starting point for this recursion must be a value for W_1, the life expectancy of an actor with no strong neighbors. Taking him as immortal, the value is $\frac{1}{1-F}$, which is the sum of the infinite series $1 + F + F^2 + \ldots$

First explorations

Now what one immediately finds in these equations is recursion in the customary sense (case $\underline{n}$ from case $\underline{n-1}$). By solving these equations for them, $\underline{W_n}$ and $\underline{S_n}$ are defined in terms of $\underline{W_{n-1}}$ and $\underline{S_{n-1}}$.

$$\text{(Eq.9)} \qquad S_n = \frac{n-P_n+P_n^2 + FP_n\left[(n-2)S_{n-1} + W_{n-1}\right]}{1 + P_n(n-2) - (1-P_n^2)F}$$

$\underline{W_n}$ may be obtained by substituting this in the first of (Eqs.8).

$\underline{P_n}$ may be treated as a variable. If, starting from a given tentative value, raising $\underline{P_n}$ a bit makes $\underline{S_n}$ increase, the strong actors, whose life expectancy $\underline{S_n}$ is, would surely be in a position to effect that increase. (No other reason against their ganging up on the weak actor was yet spelled out for the model.) Conversely, if lowering $\underline{P_n}$ a bit is what makes $\underline{S_n}$ increase, the strong actors should also be able to act more stably on that incentive.

One may see from (Eqs.8) and (Eq.9) that, starting from any value of $\underline{P_n}$, if $\underline{P_n}$ is varied in the direction which increases $\underline{S_n}$, further changes in that direction continue to increase it. One can say that all possible values (from 0 to 1) of $\underline{P_n}$ fall into two zones: an upper zone where a positive feedback re-enforces stability and a lower zone where positive feedback re-enforces instability.

This promising handle on circular re-enforcing arguments about pre-emption was notably absent from the pilot model. The natural next step was to investigate the new model with reference to $\underline{L_n}$, the critical value dividing the two zones. Surely the rise and fall of $\underline{L_n}$ would mark the greater or lesser stability of $\underline{n}$-actor systems.[10]

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[10]It does. The first calculations of L_n were really calculations about the minimal model which is latent in this model. It was, however, difficult to think about the model in this prototype form, probably

Possible Use

As a recursive model (in the usual sense of defining values for the case n from the case n-1) this model seemed to answer Kaplan's wish. Firstly, the alternation argument[11] tends to reach the same conclusion as Burns' pet idea for "a comparatively simple model of the international system,"[12] namely that odd numbers of approximately equal actors tend to make a stabler system than do even numbers of actors.

Secondly, the pre-emptive considerations of this model evidently could be made to represent pre-emptive deviancy. Kaplan's exposition[13] grants for argument that in a particular size system everyone's security rationality might indicate the stabilizing course of sparing the weak. Nevertheless, it goes on, an actor must consider that many times in the future he will be the weak. How perfectly security rational will the strong actors be then? Will they see his danger in time? Will one of them be making a grab for the extra pleasures of hegemony? With these questions posing a background risk, perhaps a strong actor should make a grab for hegemony just to survive. Clearly there is some survival-based pre-emptive motive.

Kaplan has been willing to base his pet idea regarding the optimal number of actors on the dilution of this pre-emptive motive. He

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because P_n is used in different ways. Besides looking for P_n's critical levels, $I_{\overline{n}}$, in order to make recursive calculations, it is also necessary to estimate background risks or minimal levels for P_n in the smaller systems.

[11] See page 46 above.

[12] Burns, "From Balance . . . ," apud Rosenau, Reader . . . , p.364b.

[13] See Kaplan, "Theoretical Inquiry . . . ," pp.19-39.

[14] Notice how the evaluating actor is provoked to deviance by taking a long view, cf. the second result of the minimal model, p.46, above.

preferred to find this dilution in active roles of restraint (of hegemonial deviants) and in extra actors filling the shoes of actors who have mistaken their part.[15] I have found grounds[16] of dilution in the mere numbers which are at least the proper substratum for considering dilution effected through active roles.

Technical Progress

Technical progress consisted of stripping away details, some of which only look like the easiest assumptions but are not really the simplest choice. As a result of stripping down the model its behavior became fully known and the results implicit in that behavior were more clearly seen.

The first simplification of the model was to get away from separate references to weak and strong actors. This was done by assuming that in stable transitions the weak actor should have the same chance as the strong to be weak in the next round. This way the recursion formulae were rewritten in terms of $\underline{A_n}$, the life expectancy of an average member of the $\underline{n}$-actor system.

$$\text{(Eq.10)} \qquad A_n = \frac{n-P_n + F(n-1)P_nA_{n-1}}{n\left[1 - F(1-P_n)\right]}$$

Under this change of assumption strong actors who do not pre-empt are still giving the weak actor a sizable benefit, a chance, though not a certainty, of strength.[17]

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[15]Cf. Kaplan, "Theoretical Inquiry . . . ," pp.19-39, with "The Systems Approach . . . ," p.390.

[16]The third and fourth results of the minimal model, pp.46-47 above. Numbers make the hegemonial try not worth the attempt. They also dilute the chance of being stung when one passes up pre-emption and finds that the system reduces after all.

[17]It was as if one stable round now represented a different span

A minor, but convenient alteration was to normalize these $\underline{A_n}$ (by multiplying them all by 1-F) so that the maximum value, A_1, that of sole survivorship, is equal to $\underline{1}$. It is evidently correct, in comparing values for different cases of the discount factor, to factor out the inflation of life expectancies caused by a slow discount.

Calculations were made with the prototype model thus modified for some cases (parameter values). The behavior was similar to that which finally characterizes the minimal model,[18] but it was difficult to see what was general and whether it was. Namely, recursive calculations were made of $\underline{L_n}$, the critical level of probability, interpreting it as the primary indicator of stability, and assuming both a constant level of background risk (P_n=1) and a very steep discount factor of $\frac{1}{2}$.

TABLE 2

LIFE EXPECTANCIES, A_n, AND CRITICAL LEVELS, L_n,
ABOVE WHICH A BACKGROUND RISK WILL TRIGGER PRE-EMPTION

Number (n)	Normalized Average Value (A_n)	Critical Level (L_n)
1	1.0	(meaningless)
2	.5	0
3	.909	1.0
4	.948	.19
5	.958	.1336
6	.966	.1288
7	.968	.121

The critical level was first low, which implied pre-emption. Then $\underline{L_n}$

in a process of passing the weak position around. Thinking of this process as continuous was the last simplification of the model, below p. 59.

[18] See pp.45-47 above.

rose, a non-pre-emptive case; then fell again, another non-pre-emptive case. It continued falling slowly thereafter, all non-pre-emptive cases.

Hindsight shows that the significant fact is that only the first low of $\underline{L_n}$ corresponded to a pre-emptive case, but the dip in critical level at $\underline{n}=4$ seemed to be an inclination of the model toward alternation, even though it did not come to the substantial result of pre-emption.

Investigation then proved that $\underline{L_n}$ continued falling towards the value of the background level. There seemed no ground for assuming a jog upwards in the background risk, which would trigger pre-emption, so this dampened (but did not extinguish) hopes for a continuing alternation in the model. There was compensation in getting a capsule description of the behavior at higher values of $\underline{n}$.

Variation with $\underline{n}$ was thus tentatively disposed of. Variation over $\underline{P_n}$ and $\underline{F}$ was still to be investigated. Variation over $\underline{n}$ of background levels in $\underline{P_n}$ was still a major problem. It was not yet clear that the model had been closed out with an appropriate set of explicit parameters.

Another clue to the model was the fact that background risk and discount factor worked to opposite effect. A more exact statement of their inverse equivalence was evidently desirable. One would then have essentially only one major parameter to vary in exploring the model and a conclusion about their inverse roles could be safely drawn.

The next, and most useful, change was to shift attention from critical values, $\underline{L_n}$, of the probability, $\underline{P_n}$, of instability to critical values of $\underline{A_{n-1}}$. It had been confusing to think both of minimum background levels of the probability of instability and of hypothetical rises in this probability to a level that triggered pre-emption.

There had been an illusion of a more detailed picture of the pre-emptive process that the model actually depicts.

Bringing $\underline{A_n}$ to the center also put this ambiguity in focus. The basic recursive relation[19] was being used in two ways. On the one hand it was being reused over all values of $\underline{n}$ for a long recursion back to a particular starting value (the life expectancy of the solitary survivor). On the other hand it expressed a relationship between the values for the $\underline{n}$-actor and for the $\underline{n-1}$-actor systems which holds under variation in the latter values. A relation is called "formula" when used for separate values of a variable and "transformation" when studied over a related range of values. Math teachers stress the importance of the implicit difference in approach. Transformation thinking had been applied, but at the wrong point, the $\underline{P_n}$. It was not only technically better to look at the $\underline{A_n}$ recursion as a transformation, it is theoretically best too. It usually makes more immediate sense to talk about reduction to the next size system, than about a recursive calculation over many future stages. Only in the case of a hegemonial scramble does one obviously talk with reference to the last stage.

The handle provided by a linear transformation is its fixed point, a value which is carried into the same value. Since the coefficient of the variable term[20] in this case is both positive and less than

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[19] That is, a relation for determining life expectancies in non-pre-emptive $\underline{n}$-actor systems from life expectancies in $\underline{n-1}$-actor systems. Its final version is (Eq.6), page 44 above. (Eq.9), p.52, and (Eq.10), p.54, have been the earlier versions.

[20] That is, the $\dfrac{\frac{P}{M}(n-1)}{1 + \frac{nP}{M}}$ which is multiplied with $\underline{A_{n-1}}$ in (Eq.6), p.44 above.

1, other values at any given distance from the fixed point are taken into tranform values which are on the same side of the fixed point, but nearer. This, however, means that the fixed point, call it C, is the critical value for A_{n-1}. A value of A_{n-1} from beneath C corresponds to a value of "AifNO" which is above A_{n-1}, because nearer to C. Hence, with such an A_{n-1}, the n-actor system will be non-pre-emptive. Conversely, a value of A_{n-1} above C corresponds to an "AifNO" beneath A_{n-1}, since nearer to C, hence to pre-emption in the n-actor system.

So far this is only the equivalent of what had been seen about P_n in noting the existence of L_n. The usefulness of doing all this for A_n comes in because a comparison can easily be made over successive recursions. Suppose that with increasing n the critical values (fixed points of (Eq.6) stay the same, or at least do not fall. Then it follows that alternation will not occur.

More specifically, if the n-actor system is non-pre-emptive, so is the n+1-actor system, given the main hypothesis that the critical values due to background risk do not ascend with n.

Proof: (1) Denote the critical values of n-actor and n+1-actor systems by C_n and C_{n+1}, respectively.

(2) $A_{n-1} < C_n$, because the n-actor system is hypothesized non-pre-emptive

(3) "AifNO" $< C_n$, because a positive linear transformation give values on the same side of a fixed point

(4) $A_n =$ "AifNO", because the n-actor system is non-pre-emptive

(5) $C_{n+1} \geqslant C_n$, by main hypothesis

$\therefore$ (6) $A_n < C_{n+1}$ elementary transitivity of inequalities;

but this means that the n+1-actor system is non-pre-emptive.

This, the main result of my inquiry into the minimal model, follows logically from the central improvement of looking at $\underline{A_n}$ instead of $\underline{P_n}$. Actually, it was preceded by a final improvement which tidied up a virtual redundancy.

The last change was to make the non-pre-emptive phase of the model continuous, with a risk of reduction that could occur at any time. Now, in the discrete case (action by rounds), the risks were assumed to be the same in similar successive rounds. The continuous analogue of this is an exponential probability distribution. This has the defining property that the chances of an event (reduction) first occurring in any short period of time, $\underline{dt}$, are independent of the time, $\underline{t}$, that has gone by so far.

The discount function which had increased geometrically round by round (discounting twice gives a squared discount factor), also has an exponential function as its continuous analogue. Now an actor who assesses the risks of a non-pre-emptive situation must add up (integrate) all the possible occasions on which the reduction might occur. Each is discounted appropriately. Integrating from the present to infinity removes all trace of the exponential form.

Since exponential functions multiply together by a simple rule (add the exponents) the rates of background risk and of discount were brought into that simple relation which had been expected.[21] In fact they appeared together as numerator and denominator of a fraction. Comparison of the resultant (Eq.6) with its earlier versions, Eqs. 9 and 10, marks the progress. The simple relation between risk factor and discount

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[21]Cf. p.56 above.

factor had eluded the grasp of (Eq.10). Thus, reformulation as a continuous model was a useful technical simplification. Further account of this establishment of (Eq.6) is given in note 29, page 66 below.

Theoretical Progress

Theoretical progress consisted principally of recognizing the shape of Kaplan's and Burns' ideas and finding that the minimal model could do without them.

The first thing observed was about Arthur Burns' thesis that odd numbers of equal actor make for a stabler system than do even numbers. His argument is not the alternation argument. Burns' argument[22] is essentially rather like the classical notion of trimming. He seems to hold that a balancer should be able to throw a moderately decisive weight on either side of a deadlock. He appears to assume that extremely evenly divided systems often have to resolve an issue by a ruinous war of attrition--in modern terms a thermonuclear showdown--but that a moderately preponderant coalition will be able to extract reasonable concessions from a coalition that recognizes its temporary inferiority. These conditions of non-polarization occur better in odd-numbered than in even-numbered systems of roughly equal actors.

In terms of the minimal model, these conditions of Burns' idea would effect a fluctuation over $\underline{n}$ of the background risk. Now the logic of the minimal model without that fluctuation implies that the non-pre-emptivity of larger systems is delicate. That is, the recursively computed values of $\underline{A_{n-1}}$ tend toward the critical value. If the background risk of pre-emption in a particular $\underline{n}$-actor system is now assumed to rise, the critical value is lowered and pre-emption should

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[22] Burns, "From Balance . . . ," apud Rosenau, Reader . . . , p.355b.

be triggered. Thus the minimal model, if extended, would not disappoint the obvious expectation that it reproduce Burns' arguments about alternation.

The next item of theoretical interest was the realization that Kaplan's ideas about the dilution of pre-emptive motive through numbers must also be found in a "model behind the model" showing itself in the rationale of assumptions about background risk. The pilot model of the preceding chapter is one such ulterior model, an explicitly stated one, not the most appropriate one.

The minimal model takes as an input assumption that the background risk will vary directly with the number of actors.[23] Kaplan's ideas imply two sources of that risk which may sometime be further spelled out in a model behind the model. They may be called blunders and deliberate deviancy.[24] The minimal model's assumption that these

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[23] The effect of this assumption in the minimal model exhibits it as a neutral assumption. If one assumes a background risk rate of the form nP, where $\underline{P}$ is constant, the fixed point, C_n, of that transformation (Eq.6) is constant over $\underline{n}$. Multiplying the risk rate by n evidently offsets the fact that the chance of losing out on entry into the $\underline{n-1}$-actor system is the declining fraction, $\underline{1/n}$.

[24] Deviancy is the sense of the pilot model is as much a blunder (in its effect) as a greedy aberration (in its short-range intention) and a poorish representation of either blunders or hegemonial deviancy.

It might require more modelmaking to say much about the difference between blunders and deviancy in their relation to the assumption about the growth of background risk over $\underline{n}$. It does seem plausible that individual blunders should add up (abstracting from compensatory balancing action, as the text says). Total hegemonial deviancy is most plausible in a system of critical size, i.e. the smallest non-pre-emptive system. If one provides for local rivalries which do not directly affect the whole system, one can conceive of a partial deviancy aiming at the elimination of only some members. A peripheral actor such as Turkey is an easy example of an actor whose removal might be desired by an actor (Austria) not necessarily holding universal pretensions. Non-peripheral part systems like the German Confederation should provide the most interesting generalizable example of local rivalries which stop short of total system roll-up.

risks rise with the number of actors seems to abstract from Kaplan's essential point. This was that the number of replacements for blunderers and the number of potential restrainers of deviants is a cause making larger systems stabler. The minimal model assumes essentially that any actor's misdeed has a fixed chance of destabilizing the system independently of the number of other roles. Even on this assumption so unfavorable to dilution, the minimal model retains an elementary dilution (only one chance in _n_ of successful hegemonial grab) that first stabilizes larger systems and a secondary dilution (only the declining _n_th chance of being stung) that offsets the alternation argument (if the _n-1_-actor system is so good, let's get there now). _A fortiori_, whatever can be spelled out for Kaplan's ideas about extra available coalition partners for restraint or replacement can be expected to strengthen his inference about behavior, which is already established, though tenuously, on more restricted grounds.

The pilot model itself already suggests a much shallower rise over _n_ of the background risk. In that model only one actor is in a critical position at any time. If that were the whole story the background risk would not rise over _n_ at all. That, however, is unacceptable. Writers also give reasons why numbers exacerbate[25] matters. Occasions for war might conceivably mount with the number of frontiers or bilateral relations. Since these go up with _n_ as $\frac{n(n-1)}{2}$, which is faster than _n_, such a reason, if dominant, would argue for pre-emptive reduction of the system to manageable proportions.[26] Kaplan, Burns, Quandt and I would

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[25]The sprightliest scientific account of overpopulation leading to system breakdown is E. Deevey, "The Hare and the Haruspex: A Cautionary Tale," _American Scientist_ (September, 1960), pp.415-430. It should be taken seriously.

[26]Deutsch and Singer, p.405, with strange sanguinity draw the

say that such reasons do dominate with large numbers. Accordingly an upper limit is expected to the (optimal) number of actors for a stable system. Burns' strategic reason why complexity destabilizes a system seems weightier than the mere multiplication of channels of communication and control. (After all, Consuls and Area Desks of the State Department will probably increase in proportion to the solid interests being monitored.) His reason is the uncertainty of calculations about other Powers' reactions to one's own commitments. There is a place where this might fit into the previous chapter's pilot model. In the blind bargaining subgame of that model, actors attained to an unrealistic certainty about their fellows' reactions. (This led to the artificial result of actors not waiting for an agreement but striking unilaterally.) There must be a plausible rationale for discounting values which rest on chains of such diplomatic predictions and this[27] will give a representation to Burns' ideas about uncertainty.

Technical Details

Obviated Calculation of Tables of Behavior

Proceeding from definitions, a piecemeal exploration of the minimal model is made by recursive calculation of $\underline{A_n}$ for given values of the parameter ratio P/M. Starting from a value of $\underline{A_{n-1}}$, initially

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opposite conclusion from complexity and find more dilution. It is as if diplomatic interchanges were the cause, not the palliative, for war. The maxim "Let sleeping dogs lie" is inappropriate if serious conflicts of interests are postulated for the system. If, on the other hand, enlarging the system is a nominal inclusion of distant actors who are really out of sight, only then would it be safe to leave them out of mind, but this is not an interesting approach.

[27]A certain plausible revision of the pilot model would make military action proceed simultaneously with bargaining in continuous time. Such a model could be the basis for a synthesis of both the present chapters.

$A_1=1$, one asks if it is greater than C. If so, the value of "AifYES" is taken from (Eq.7) as the value for A_n and pre-emptivity is noted. If A_{n-1} does not exceed C then A_{n-1} will not exceed the value "AifNO" taken from (Eq.6), page 44 above, as the argument on pages 57-58 above shows. Accordingly, that is the value of A_n and non-pre-emptivity is noted.

Proceeding this way tables of behavior similar to Table 2, page 55 above, can be generated. If graphed, the values of A_n will show an initial plunge (pre-emptivity) through values $1/n$ until they fall to or below the critical level, $C = 1/\ 1+\frac{P}{M}$. When A_{n-1} is not above this level, the n-actor system is non-pre-emptive. If calculations and graphs were protracted they would show that A_n asymptotically reapproaches C from below.

The theorem against alternation, page 58 above, proves this, under the assumption of background risk, nP,[28] thus obviating the calculation, which could only now be an illustration.

This establishes the final capsule statement of the minimal model behavior given on page 45 above. The results seen from that behavior are largely self-explanatory. Only one detail of any interest seems to resist translation from technical language.

The Effect of Secondary Dilution

This is how secondary dilution "just" manages to overcome the pull of the alternation argument. That there is an overcoming follows from the result that there is no alternation in the non-pre-emptive tail

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[28] This assumption is what permits reference to a C constant over n. If the background risk increases more slowly than nP, there is an ascending C_n and the theorem against alternation still holds, together with all consequences. If the rate rise more sharply than nP, C falls and there can be alternation. If there is a sufficient jog, and after A_n has approached C closely a small jog is sufficient, there will be alternation, *viz.* pre-emption in a larger system.

of the sequence of values A_n. That it is tenuous may be seen from the fact that later values of A_n crowd the critical value C (from beneath).

It is useful to consider an even narrower gap than that between a value of A_{n-1} and C. I name as anti-pre-emptive gap, G_n, the difference between "AifNO" and A_{n-1}. (Since "AifNO" becomes the value of A_n it is readily seen how the sequence of all subsequent G_n are terms adding up to the whole gap from A_{n-1} to C.)

(Eq.11) $\quad G_n + A_{n-1} = \text{"AifNO"}$

There is another way of expressing G as an anti-pre-emptive gap.

(Eq.12) $\quad G_n + H(A_{n-1})(\frac{1}{n}) = (1-H)(1-A_{n-1})$, where $H = \frac{nP}{M+nP} = \frac{nP/M}{1+nP/M}$.

This can be proved[29] algebraically by substituting (Eq.6) in (Eq.11).

(Eq.12) introduced five new factors. The two on the right hand side are the 1-complements of the first two factors on the left hand side.

I assert that, with increasing n, only the diminution of the third factor tends to keep the gap, G_n, open and the system non-pre-

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[29] It can also be established from the ground up. To do this H must be explained as an averaged discount factor. Averaging is taken over an exponential probability distribution, $\exp(-nPt)$, which gives the fraction of n-actor systems which survive the background risk rate nP to the time, t. What is averaged is the typical actor's discount factor $\exp(-Mt)$ which applies for events deferred to time t. The resultant integrated formula for H gives an averaged discount factor appropriate to events contingent on the n-actor system's reduction because of background risk. The complementary factor, $1-H$, is appropriate to a lease enjoyed until that reduction. (If $1-H$ is for a lease, the legal antonym remainder applies to H.) This rationale actually underlies (Eq.6) where actors in a non-pre-emptive system enjoy a lease on simple existence (value 1) plus a remainder on the deferred "AifYES."

Applying the rationale to (Eq.12) it may be interpreted as comparing the cost paid by and the benefit to an actor who chooses non-pre-emption over pre-emption. The cost, on the left, is the averaged discounted nth chance of losing one's place in the $n-1$-state when deferred reduction occurs; a pre-empter could have secured membership at the outset. The benefit is a lease on simple existence rather than the chances of membership in the $n-1$-state, the lease running until the reduction occurs.

emptive. The other factors tend to close and reverse the gap.

The factor $(\frac{1}{n})$ evidently contributes to the anti-pre-emptive gap because it lowers the left hand member. The growth of $\underline{A_{n-1}}$, however, both raises the left member and depresses the right member, in each case nibbling away at the anti-pre-emptive gap. (This is the alternation argument.)

$\underline{H}$ also grows with $\underline{n}$, as is seen by examining its formula. The growth of $\underline{H}$ may be traced back to the choice of $\underline{nP}$ to give individual actors a constant background risk rate; larger non-pre-emptive $\underline{n}$-states reduce faster.

At the beginning of the non-pre-emptive tail of the table of behavior, primary dilution has given an initial cushion towards non-pre-emption, which shows up as a more or less large positive $\underline{G_n}$ (the $\underline{n}$ in point is the size of the first non-pre-emptive system). Even this initial cushion can be made small by choosing a parameter ratio P/M slightly less than an integer. In any case the asymptotic approach of $\underline{A_n}$ towards $\underline{C}$ consumes the initial capital of the anti-pre-emptive gap. The declining factor $(\frac{1}{n})$ is the sole factor which makes new contributions against pre-emption. The other factors draw on those contributions.

Following the rationale given at the end of the preceding footnote the three-factor term in the left member of (Eq.12) is seen to express the cost of being stung, not having pre-empted in an $\underline{n}$-actor system, and especially the secondary dilution comprised in the fact that this risk is shared among $\underline{n}$ members. I have thus shown how, with increasing $\underline{n}$, this secondary dilution overcomes the pull of the alternation argument (as well as the pre-emptive motives attributable to the assumed accelerating background risk of reduction).

CONCLUSION

The conclusion of this dissertation is that realization-models are a feasible tool for exploring the idea of the "balance of power."

A realization-model is an artificial laboratory world created for the sake of exploring formal theory. This exploration can be by means of controlled experiments, inquiries about "what if . . . instead of . . . ?" Counterfactual or "unreasonable" assumptions can be made and their consequences traced.

For example, the pilot model imputed to actors an objective prediction of others' diplomatic responses to their several possible commitments, since a subtler rationale did not come to mind. A consequence of this imputation was the artificial result that actors make all their moves unilaterally, for the reason that a promise need not be sought from someone whose behavior is quite predictable.

Now it seems possible to make a better mathematical picture of the uncertainties pinned down by a multilateral agreement. This picture will probably illuminate what Arthur Burns points to when he speaks of the "uncertainty produced by multiplication of decisions"[1] as quickly setting an upper limit to the optimal number of actors for stability.

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[1]Burns, "From Balance . . . ," apud Rosenau, Reader . . . , p.355b.

Unlike simulation models which are supposed to contain substantial amounts of all the theory they represent, realization-models leave out, as far as possible, those details which cannot be given a good workout in a particular application. They are sketches rather than master plans of theory. This preference for simple models seems justified by the results. The minimal model proves nearly as much as the pilot computer model, yet the minimal model is a simple paper and pencil calculation based on five variables, by a reasonable count.[2]

A great justification of the simplicity of these models is that simplification can be undone fairly easily and without losing track of old and new shapes. Mathematical realization-models are like modular furniture. They can be expanded and compressed according to their application. Thus, the models in chapters i and ii are intimately related and this relationship, being a matter of definite mathematical forms, can be elaborated as much as one could want. On the one hand, description of the minimal model required only a brief account of the way in which it simplified the pilot model. On the other hand, the just mentioned attack on Burns' ideas on how "uncertainty and momentousness of decisions"[3] limit stability in large systems is expected to use a synthesis of the two models. It will probably imply a more thorough examination of their relation.

The substantive results of these chapters are to be taken rather as examples of how one might argue than as assertions proposed

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[2]Time; number of actors; pre-emptivity (which is a mere yes/no); life expectancy (which is logically a derived quantity, but is juggled independently in the argument); the ration of background risk rate to discount rate (two parameters thus appearing essentially as one).

[3]Burns, loc.cit.

for empirical test. More or less plausible verbal arguments might be found to qualify or to reverse any of these conclusions. Mathematical modeling has to be able to imitate such arguments and expound their assumptions with greater clarity. It proves no synthetic truths, instead it helps one to analyze for the assumptions.

I include as substantive even a result so merely formal as dynamic stability because it is an example of a range finding result. With mathematical models all results can be treated profitably as range finding results, again for the reason that these are such definite means of incorporating qualifying assumptions. The classic idea of the "balance of power" would seem to include a restless round of wars caused by the balancing motives themselves.[4]

The main result of the first chapter, that long range balance does not necessarily imply evenly matched wars, amounts to restoring the classic emphasis on the Grand Coalition as the normal means of a "balance of power" system.[5]

The point of that result in this dissertation is that it was drawn by mathematical means from initial assumptions apparently otherwise inclined. Verbal theorizing, being easier, is less likely to draw all the complex behavior possible from simple assumptions.

Other, abortive results, such as those sensitive to undigested features of the pilot model's bargaining algorithm, do have potential

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[4]E. Haas, "The Balance of Power: Prescription, Concept, or Propaganda," apud Rosenau, International Politics . . . , p.324. This nuance is clearly to be seen from the use of the idea "Power Politics" as a bad name, cf. H. Morgenthau, Politics Among Nations (New York: Alfred Knopf, 1948), p.37.

[5]Cf. Hume, "On the Balance of Power," Essays . . . ,pp.201-203, and W. Churchill, Marlborough: His Life and Times (London: George C. Harrap and Co., Ltd., 1947).

merit. If a mathematical model is patched up, the origin of the salvaged parts can be recognized. With words, however, it is hard to say when and where a useful idea is first spelled out.

The prospect (not an execution) of rearranging utility theory points to the kind of mathematical theory building to be desired for political science. Presently, verbal theory and ordinary language must be kept as a guide.

The pilot model was built first and questioned after. The minimal model is primarily important as an example of a model used to hunt down already scented theoretical ideas. The result most directly sought from it is that under certain most elementary assumptions there is not a pre-emptive/non-pre-emptive alternation of stability and instability for successive numbers of actors. An additional condition about a self-fulfilling fear of pre-emption will make the alternation argument work.

The sting of guessing wrong about pre-emption is shared among all the actors, hence diluted by increasing numbers. This is what just overcomes the pull of the alternation argument; to wit, that actors in a larger system should pre-empt to gain a non-pre-emptively stable next smaller system. How dilution "just overcomes" alternation seems best left in the mathematical form. I think it does not readily translate into words. Till something meatier comes along, this argument may be taken as a paradigm of arguments which have to be left in technical form.[6]

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[6]Since theory overall is still at a simple level, this particular necessity of technical formulation must be discounted an accidental consequence of the concepts chosen. It will take a lot more work to make a point which substantially requires mathematical language.

The making of the minimal model also served the intended purpose of clarifying Kaplan's ideas about the dilution of pre-emptive deviancy and some of Burns' ideas.

The minimal model itself fulfills a half-intended purpose of representing not so much Kaplan's as the minimum strategic aspect of dilution. The more actors, the less chance a grand try for sole survivorship has of succeeding. This banality may be taken as a bedrock reason why there is safety in numbers. The secondary kind of dilution has already been referred to in the shared chance of guessing wrong about the pre-emptivity of a single round.

The model contains essentially two reasons why the lower limit of stability might be pushed beyond three or even four actors. A high background risk of reduction in non-pre-emptive systems is an obvious reason from the start. Less obvious is the equivalent subjective effect of taking a long view. Actors with a slow discount rate for future events see a future containing many risks and pre-empt accordingly. It was appropriate that the minimal model should represent this equivalence by a simple ratio, since no useful qualifications have been elaborated. Thus the minimal model was technically superior to its prototype. Much of the work in mathematical models is paring them down to a simplicity appropriate to the ideas one can actually put in them.

In fine, the foreseeable use of mathematical models for verbal theories, such as one can now make about the "balance of power," is that they help one to make and unmake arguments the hard way.

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